Navier-Stokes equations with directional do-nothing boundary conditions

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Many engineering applications involve fluid flow in complex and large domains, making their numerical treatment too computationally expensive or even practically impossible. In order to avoid these implementation drawbacks, domain truncation is a typical procedure in mathematical and numerical modeling. This approach, however, raises the question how to define proper boundary conditions at the artificial boundaries of the computational domain, in order to reproduce the nature of the original physical problem.

In the case of incompressible Navier-Stokes flow, a classical artificial boundary condition is the so-called do-nothing (CDN) condition. Nevertheless, the CDN boundary condition generates stability issues that may compromise existence and computation of solutions for the Navier-Stokes (NS) equations. Consequently, an alternative Neumann boundary condition for the NS equations, nowadays referred to as directional do-nothing (DDN) condition, has been proposed by C.-H. Bruneau and P. Fabrie and further exploited by many other authors.

In this talk, we consider the steady Navier-Stokes equations with mixed boundary conditions where the DDN condition or a regularized directional do-nothing (RDDN) condition are imposed on the Neumann portion of the boundary. The DDN boundary condition is defined in terms of a max-function, which is not differentiable in the classical sense. We will discuss the difficulties one faces when trying to apply the Newton method to numerically solve the NS-DDN system and in describing the first order necessary optimality conditions in terms of dual variables when solving a velocity tracking problem for the same system.

This is a joint work with Pedro Mendes Nogueira and Jorge Tiago from the Department of Mathematics and CEMAT, Instituto Superior Técnico, Universidade de Lisboa, Portugal.