

# Existence and stability of equilibria of free-falling plates

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Steady gliding and diving motions in animals such as birds may be modeled by considering thin flat plates falling passively through fluid. We investigate equilibrium solutions of an existing quasi-steady two-dimensional nonlinear model of thin rectangular plates subject to gravitational and fluidic forces at intermediate Reynolds numbers. We first demonstrate geometrically the existence and uniqueness of such equilibria for a set of fixed dimensionless parameters. We then examine a broad range of these equilibria through linear stability analysis, and present phase diagrams revealing a highly complex structure of stable and unstable regions including multiple Hopf bifurcation boundaries. We verify these findings via the full nonlinear model, and highlight some connections to unsteady flight modes. We propose a necessary condition for glider stability based on the sign of the derivative of the aerodynamic center of pressure, alongside other contributing factors for stability. We also identify a necessary condition for the stability of divers. Notably, we uncover a new distinct flight mode, emerging from a limit cycle, that supplements those previously documented. These results offer broad generalization to flat plates with arbitrary geometry and density, as well as to fluids of varying densities. We conclude by highlighting some connections to real examples of steady flight observed in nature.