Steady compressible Navier–Stokes–Fourier system with temperature dependent viscosities

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We consider the model describing the steady flow of a compressible heat conducting fluid in a bounded three-dimensional domain

$$\operatorname{div}(\varrho\mathbf{u}) = 0,$$
$$\operatorname{div}(\varrho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbf{S} + \nabla p = \varrho\mathbf{f},$$
$$\operatorname{div}(\varrho E\mathbf{u}) = \varrho\mathbf{f}\cdot\mathbf{u} - \operatorname{div}(p\mathbf{u}) + \operatorname{div}(\mathbf{S}\mathbf{u}) - \operatorname{div}\mathbf{q}$$

with ϱ the density, $\mathbf{u}$ the velocity field, $\mathbf{S}$ the stress tensor (here we assume the fluid to be Newtonian with temperature dependent viscosity), p the pressure, $\mathbf{f}$ the given volume force, $\mathbf{q}$ the heat flux and the total energy $E = \frac{1}{2}|\mathbf{u}|^2 + e$ with e the internal energy. We assume the pressure law of the form $p(\varrho,\vartheta) \sim \varrho^\gamma + \varrho\vartheta$ with $\gamma > 1$ and the viscosities $\mu(\vartheta), \xi(\vartheta) \sim (1+\vartheta)^\alpha$, $\alpha \in [0,1]$.

We show the existence of a weak or variational entropy solution for the above model with internal energy fulfilling the Gibbs relation and the heat flux fulfilling the Fourier law $\mathbf{q} \sim (1+\vartheta)^m\nabla\vartheta$ with ϑ the temperature, $m = m(\gamma,\alpha) > 0$.

We first review the results for the case $\alpha = 1$ from papers [1], [3] and [4] which where created in collaboration with French co-authors. Then we show extensions for $\alpha < 1$. First we concentrate on the case $\gamma > \frac{3}{2}$ both for Dirichlet and Navier boundary conditions for the velocity and Robin (we extend the results from [2]) and Dirichlet boundary conditions for the temperature. Finally we briefly touch the situation when $\gamma \le \frac{3}{2}$, here only for Robin boundary conditions for the temperature. The solutions are constructed for arbitrarily large sufficiently integrable data.

It is a joint work with Ondřej Kreml, Tomasz Piasecki and Emil Skříšovský.

References

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