Stability and optimal temporal decay result for the 3D Boussinesq equations with horizontal dissipation in anisotropic Sobolev spaces

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In this paper, we study the following 3D Boussinesq equations with partial dissipations:

$$\begin{aligned} \partial_t u + u \cdot \nabla u =& - \nabla P + \nu \partial_{11} u + \nu \partial_{22} u + \Theta e_3, \\ \partial_t \Theta + u \cdot \nabla \Theta =& \mu \partial_{11} \Theta + \mu \partial_{22} \Theta, \\ \nabla \cdot u =& 0, \end{aligned} \tag{1}$$

where $u = (u_1, u_2, u_3)$ is the velocity field, P is the pressure, Θ is the temperature, $e_3 = (0, 0, 1)$ is the vertical unit vector and $\nu, \mu > 0$ are the viscosity and the thermal diffusivity, respectively.

We prove that, for the perturbed equations, the time global solution exists for small initial data in the anisotropic Sobolev spaces $H^{0,s}$ with $\frac{1}{2} < s$ and the corresponding solution of the unperturbed equations approaches the hydrostatic equilibrium. Moreover, the optimal decay result is obtained in the anisotropic Sobolev spaces $H^{0,s}$, extending the result of isotropic Sobolev spaces.

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