On self-propulsion by oscillations in a viscous liquid

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Suppose that a body $\mathcal{B}$ can move by translatory motion with velocity $\boldsymbol{\gamma}$ in an otherwise quiescent Navier-Stokes liquid, $\mathcal{L}$, filling the entire space outside $\mathcal{B}$. Denote by $\Omega = \Omega(t)$, $t \in \mathbb{R}$, the one-parameter family of bounded, sufficiently smooth domains of $\mathbb{R}^3$, each one representing the configuration of $\mathcal{B}$ at time t with respect to a frame with the origin at the center of mass G and axes parallel to those of an inertial frame. We assume that there are no external forces acting on the coupled system $\mathcal{S} := \mathcal{B} + \mathcal{L}$ and that the only driving mechanism is a prescribed change in shape of Ω with time. The self-propulsion problem that we would like to address can be thus qualitatively formulated as follows. Suppose that $\mathcal{B}$ changes its shape in a given time-periodic fashion, namely, $\Omega(t + T) = \Omega(t)$, for some $T > 0$ and all $t \in \mathbb{R}$. Then, find necessary and sufficient conditions on the map $t \mapsto \Omega(t)$ securing that $\mathcal{B}$ self-propels, that is, G covers any given finite distance in a finite time. We show that this problem is solvable, in a suitable function class. Moreover, we provide examples where the propelling velocity of $\mathcal{B}$ is explicitly evaluated in terms of the physical parameters and the frequency of oscillations.