On $L^2(0, T; L^\infty(\Omega))$ estimates of finite energy solutions to the Navier-Stokes equations in $\mathbb{R}^2$

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The results presented in this talk are jointly obtained with *Yoshikazu Giga, The University of Tokyo*, see [1].

We consider the Navier-Stokes equations in a smooth bounded domain $\Omega \subset \mathbb{R}^2$ under the no-slip boundary condition with initial velocity u_0 of finite kinetic energy, *i.e.*, $u_0 \in L^2_\sigma$, and prove the existence of a unique weak solution $u \in L^2(0, T; L^\infty(\Omega))$ satisfying the estimate

$$\|u\|_{L^2(0,T;L^\infty(\Omega))} \leq c\,(1 + \|u_0\|_{L^2})\,\|u_0\|_{L^2}$$

with some constant c depending only on Ω.

Note that $H^1(\Omega)$ is not embedded into $L^\infty(\Omega)$ so that the argument is not based on the energy equality valid for u, but on a generalized Marcinkiewicz interpolation theorem. This estimate is extended to mild solutions of Serrin's class in $\mathbb{R}^n$ provided that $u_0 \in L^n_\sigma$.

References

[1] R. Farwig, Y. Giga: *On square-in-time integrability of the maximum norm of a finite energy solution to the planar Navier-Stokes equations.* Algebra i Analiz 36, no. 3 (2024), 289-307