On the large time asymptotics of global solutions to the Vlasov-Navier-Stokes equations in the whole space

Raphaël Danchin

Université Paris-Est Créteil, France
danchin@u-pec.fr

The incompressible Vlasov-Navier-Stokes equations are a toy model for describing the dynamics of a cloud of particles that are immersed in an incompressible viscous fluid. Here we are concerned with the behavior of global strong solutions when the time goes to infinity, in the case where the equations are posed in the whole space. For small enough initial data with sufficient integrability at infinity, it is shown that the velocity decays to zero, with the same rate as the classical Navier-Stokes equations, and that the kinetic distribution of the particles is well approximated by a monokinetic distribution with velocity which is the same as that of the viscous fluid. We first establish this result in the 'smooth' case of H^1 velocity fields, then show that it remains true for critical regularity $H^{1/2}$.

Our results rely on the use of a higher order energy functional that controls the regularity H^1 of the velocity and seems to have been first introduced by Li, Shou and Zhang in [12] in the context of the nonhomogeneous Vlasov-Navier-Stokes system, and on the propagation of suitable negative regularity.