On some recent results from the theory of the MHD equations

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We consider the system of the magneto-hydro-dynamic (shortly MHD) equations for the unknowns velocity $\mathbf{u}$, magnetic field $\mathbf{b}$ and pressure p in $Q_T := \Omega \times (0,T)$, where Ω is a "smooth" domain in $\mathbb{R}^3$ and $T > 0$. We assume that both $\mathbf{u}$ and $\mathbf{b}$ satisfy the so called Navier–type boundary conditions $\mathbf{u} \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} = 0$ and $\mathbf{curl}\,\mathbf{u} \times \mathbf{n} = \mathbf{curl}\,\mathbf{b} \times \mathbf{n} = \mathbf{0}$ on $\partial\Omega \times (0,T)$.

We assume that Φ is a Young function, that, in addition to the properties directly following from the definition, also satisfies the conditions (i) $s^{-3/2}\,\Phi(s)$ is monotone increasing on $[0,\infty)$ and tends to infinity as $s \to \infty$, (ii) $\Phi(s)^{-2/3} \in L^1((1,\infty))$.

The main result (see [1] and [2]) roughly speaking says that if Ω' is a sub-domain of Ω and $0 \le T_1 < T_2 \le T$ then the vector fields $\mathbf{u}$ and $\mathbf{b}$ are Hölder continuous in $\Omega' \times (T_1,T_2)$ if at least one of the following conditions holds;

1) $p_- \in L^\infty(T_1,T_2;\, L^\Phi(\Omega'))$,

2) $\mathcal{B}_+ \in L^\infty(T_1,T_2;\, L^\Phi(\Omega'))$, where $\mathcal{B} := p + \frac{1}{2}|\mathbf{u}|^2 + \frac{1}{2}|\mathbf{b}|^2$.

Here, p_- denotes the negative part of p, $\mathcal{B} := p + \frac{1}{2}|\mathbf{u}|^2 + \frac{1}{2}|\mathbf{b}|^2$, $\mathcal{B}_+$ denotes the positive part of $\mathcal{B}$ and $L^\Phi(\Omega')$ is the Orlicz space.

This is a joint work with Minsuk Yang from Yonsei University, Seoul, Republic of Korea.

References

[1] J. Neustupa, M. Yang, New regularity criteria for weak solutions to the MHD equations in terms of an associated pressure, *J. Math. Fluid Mech.* **23** (2021), Article no. 73, 24 pp., online https://doi.org/10.1007/s00021-021-00597-9.

[2] H. J. Choe, J. Neustupa, M. Yang: Improved regularity criteria for the MHD equations in terms of pressure using an Orlicz norm. *Applied Mathematics Letters* **132**, October 2022, Paper No. 108121.