Interactions of fluids with poro-elastic media – Derivation of a system of coupled Stokes and plate equations for fluid flow through a thin porous elastic layer

Willi Jäger
IWR, University of Heidelberg, Germany
wjaeger@iwr.uni-heidelberg.de

The investigation of fluid structure interaction combined with diffusion, transport and chemical reactions in porous elastic media is a highly topical subject of mathematical and computational research. In this lecture we consider particular systems consisting of free fluid flow in compartments separated by porous, elastic thin layers. This situation occurs in many application areas: soil physics, biosciences, medicine, … Here we restrict ourselves just to the fluid-structure interaction, the coupling of the homogenization of the porous structure in the layer and the reduction of the layer to an interface. We treat the linearized models for the flow and elasticity, reducing the equations for dynamics of the fluid and the elastic structure to fixed reference domains. We assume that the porous structure is periodic. Despite of these reductions, more extensive tools of multiscale analysis are needed to perform a 2-scale limit with respect to a properly chosen scale parameter ε, controlling the thickness of the layer and the periodicity of the pore structure, and to derive an effective equation for the limits.
The effective equations consist of the Stokes equation for the 2-scale limit velocity v_0 in the fluid compartments $\Omega_\pm$ and a 2-scale limit displacement u_0 of the elastic solid, defined on the interface. In the considered geometry, u_0^1 and u_0^2 vanish and u_0^3 satisfies a homogenized plate equation with the jump of the normal stress of the fluid on the solid as additional force term. v_0 is defined on the interface and equal to $(0,0, \partial_t u_0^3)$.
This result is a first step that should be followed by the determination of the term following in an ε - development to capture fluxes through the porous layer. This required analysis is in progress.

This lecture is based on joint research with Markus Gahn (Heidelberg) and Maria Neuss-Radu (Erlangen) (1) and refers to new methodological, more generally interesting results of multiscale analysis (2), which Maria Neuss-Radu will present in a further lecture.

(1) Gahn, M., Jäger, W., and Neuss-Radu, M.
Derivation of a system of coupled Stokes and plate equations for fluid flow through a thin porous elastic layer. Applicable Analysis (Andro Mikelic Memorial Volume), online June 2022 (DOI:10.1080/00036811.2022.2080673)
(Preprint: arXiv:2112.03583).

(2) Gahn, M., Jäger, W., and Neuss-Radu, M.
Two-scale tools for homogenization and dimension reduction of perforated thin layers: Extensions, Korn-inequalities, and two-scale compactness of scale-dependent sets in Sobolev spaces.
Submitted (Preprint: arXiv:2112.00559).