The Navier-Stokes Equations in Domains with Moving Boundaries

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Consider the instationary (Navier-)Stokes system on a family of t-dependent bounded or exterior domains, $\Omega(t)$, with moving boundary and Dirichlet boundary conditions. The main problem in the construction of local/global and even t-periodic solutions arises from a t-depending volume-preserving change of coordinates, $\phi(t)$, by which the system is transformed to a non-autonomous Stokes/Navier-Stokes system of the form

$$\partial_t u(t) + A(t)u(t) = P(t)F - P(t)u \cdot \nabla^{\phi(t)} u, \quad u(0) = u_0,$$

with a t-dependent modified Stokes operator

$$A(t) = P(t)\Big(- \Delta - \sum\nolimits_{|\alpha|\le 2} a_\alpha(x,t)\partial^\alpha + \sum\nolimits_{|\beta|\le 1} b_\beta(x,t)\partial^\beta \Big)$$

on $L^q_\sigma(\Omega_0)$ in a fixed reference domain Ω_0; here, $P(t)$ is a modified Helmholtz projection.

The analysis of this problem started with the L^2-theory by Miyakawa and Teramoto in the 80ies and local maximal regularity results in L^q of J. Saal (2006). This local theory was extended in [1] to global solutions for exterior domains under suitable smallness assumptions. In [2] it was shown by a uniform *BIP* property that Sobolev embedding constants and the characterization of domains of fractional powers of $A(t)$ do not depend on t. This property was the basis for the existence of mild time-periodic solutions in L^q for bounded domains with a periodically moving boundary [3]. Finally, mild solutions are shown to be regular via a Fujita-Kato iteration adapted to the non-autonomous setting [4].

In this talk we present these results and first steps to deal with t-periodic solutions for unbounded domains. The results are based on joint papers with K. Tsuda (Kyushu Sangyo University, Fukuoka), and with H. Kozono (Waseda University, Tohoku University), D. Wegmann (TU Darmstadt).

References

[1] R. Farwig, H. Kozono, D. Wegmann: *Maximal Regularity of the Stokes Operator in an Exterior Domain with Moving Boundary and Application to the Navier-Stokes Equations.* Math. Ann. 375, 949-972 (2019)

[2] R. Farwig, K. Tsuda: *Uniform Estimates for Fractional Operators.* SN Partial Differ. Equ. Appl. 10, 2:27 (2021)

[3] R. Farwig, H. Kozono, K. Tsuda, D. Wegmann: *The Time Periodic Problem of the Navier-Stokes Equations in a Bounded Domain with Moving Boundary.* Nonlinear Analysis: Real World Applications 61, 103339 (2021)

[4] R. Farwig, K. Tsuda: *The Fujita-Kato Approach for the Navier-Stokes Equations with Moving Boundary and its Application.* J. Math. Fluid Mech. 24:77 (2022)