Vortex Rings and Binormal Curvature Flow

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We present a model of vortex-ring dynamics based on motion of closed non-intersecting curves in space by curvature in the binormal direction and on their mutual interaction. In this approach, a vortex ring is represented as a closed curve in space by its core which represents concentrated vorticity and generates motion and force interaction in its vicinity. A comprehensive review on vortex-rings research can be found in Meleshko *et al.* [1]. Vortex structures can be relatively stable in time, can interact each with other and exhibit interesting dynamics.

Dynamics of vortex rings is one of important examples of the curve motion by curvature in the binormal direction (see e.g. [2]). According to [3], such a motion is investigated by means of a system of geometric evolution equations describing the evolution of a family of space curves in the binormal and normal directions. Evolving curves can interact with each other locally or nonlocally, the diffusive and redistribution effects act in the normal direction. The evolution of curves $\Gamma^i, i = 1, \ldots, n$ is treated parametrically by the corresponding geometric motion law

$$\partial_t \mathbf{X}^i = v_N^i \mathbf{N^i} + v_B^i \mathbf{B^i} + v_T^i \mathbf{T}^i, \quad i = 1, \ldots, n, \tag{1}$$

where the unit tangent $\mathbf{T}^i$, normal $\mathbf{N^i}$ and binormal $\mathbf{B^i}$ vectors form the Frenet frame, and $\mathbf{X}^i : S^1 \times [0, \infty) \to \mathbb{R}^3$ are the parametrizations of Γ_t^i. The functions v_N^i, v_B^i, v_T^i depend on the position vector $\mathbf{X}^i \in \mathbb{R}^3$, the curvature κ^i, the torsion τ^i, and on all parametrized curves $\Gamma^i, i = 1, \ldots, n$.

In the contribution, we introduce suitable tools of differential geometry and nonlinear PDE's and formulate the general motion law whose analytical properties are briefly discussed. The finite-volume scheme allows to solve (1) numerically. Nontrivial tangential velocity serves for the redistribution of discretization nodes stabilizing the numerical scheme. We demonstrate behavior of the solution on several computational studies of the flow combining the normal and binormal velocity and considering nonlocal interactions of moving vortex rings.

[1] V. V. Meleshko, A. A. Gourjii, and T. S. Krasnopolskaya, *Vortex rings: History and state of the art*, J. Math. Sci., 187(6) (2012), pp. 772–808.

[2] R. L. Jerrard and D. Smets, *On the motion of a curve by its binormal curvature*, J. Eur. Math. Soc., 017(6) (2015), pp. 1487-1515.

[3] M. Beneš, M. Kolář and D. Ševčovič, *Qualitative and Numerical Aspects of a Motion of a Family of Interacting Curves in Space*, SIAM J. Appl. Math. **82**(2) (2022), 10.1137/21M1417181.